International Journal of Theoretical Physics, Vol. 15, No. 1 (1976), pp. 1-9

Neutral Current Effects in Mϕller Scattering

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Received: 25 September 1974

1. *Introduction*

As is well known (Marshak *et al.*, 1969), in conventional $V - A$ theories of weak interactions, one considers only charged weak currents and charged intermediate vector bosons. Various arguments, both theoretical and experimental, are used to exclude weak neutral currents (WNC) and the neutral intermediate vector or scalar bosons. For example, it is argued that if WNC existed some purely weak interaction processes, such as the following, should be observed.

$$e^- + \nu_e(\bar{\nu}_e) \to e^- + \nu_e(\bar{\nu}_e) \tag{1.1a}$$

$$\mu^- + \nu_\mu(\bar{\nu}_\mu) \to \mu^- + \nu_\mu(\bar{\nu}_\mu) \tag{1.1b}$$

$$\nu_\mu(\bar{\nu}_\mu) + N \to \nu_\mu(\bar{\nu}_\mu) + N \tag{1.1c}$$

$$\nu + N \to \nu + X \text{ (hadron)} \tag{1.1d}$$

$$\nu + (Z) \to \nu + \mu + \bar{\mu} + (Z) \tag{1.1e}$$

For a long time none of these crucial reactions was observed and this tended to support the view that WNC did not exist. Support for WNC has, however, recently come from two independent sources, one experimental and the other theoretical.

Experimentally, evidence has been found (Hasert *et al.*, 1973; Benvenuti *et al.*, 1974) for reactions (1.1c) and (1.1d) in which neutrinos are coupled to hadrons. This led immediately to the renewed speculation that other WNC processes might soon be found and that WNC do indeed exist. This question has to be settled by carrying out more systematic searches for other WNC processes of the type listed above.

If one admits the idea of a WNC, one can think of another class of processes, conventionally treated as purely electromagnetic (neutral) current processes, which would now be admixtures of weak and electromagnetic interactions. The result is that we have further means of searching for WNC. Among reactions in the latter group we have the following:

$$e^- + e^- \to e^- + e^- \tag{1.2a}$$

$$e^- + e^- \to \mu^- + \mu^- \tag{1.2b}$$

$$e^- + e^+ \to e^- + e^+ \tag{1.2c}$$

$$e^- + e^+ \to \mu^- + \mu^+ \tag{1.2d}$$

$$e^+ + e^- \to \text{hadrons} \tag{1.2e}$$

$$e^- + p \to e^- + p \tag{1.2f}$$

$$e^- + p \to e^- + X \tag{1.2g}$$

Searches for WNC in these processes consists of looking for modifications to conventional electromagnetic theory results for these processes arising from the WNC contributions. From an experimental point of view one can look for corrections to the magnitude of differential cross-sections. This however involves measuring the cross sections to very high precision, beyond the tested limits of QED, including radiative corrections. This is hard to achieve.

A more promising line of attack is to look for parity-violating effects since the weak contribution will violate parity but not the electromagnetic part. The interference between the two contributions can be projected out and its effect measured by looking for forward–backward asymmetry in the differential cross-section. In addition, one can measure the helicity of the scattered electrons by measuring, for example, the angular distribution of the bremsstrahlung emitted by the scattered electrons in matter. In a two-body process such as the Møller scattering, with initially unpolarised electrons, such mean helicities should be zero if WNC parity-violating interactions are absent.

However, even when one knows what to measure in order to detect the presence of WNC, one still has to make a judicious choice of the reaction. Some processes are kinematically more favoured than others for the purpose of searching for WNC effects. Since colliding e^+e^- beams are already available, providing large centre of mass energies which may not be too far from the intermediate vector-meson (Z^0) pole, considerable effort (Cung *et al.*, 1972; Love, 1972; Godine & Hankey, 1972; Brown *et al.*, 1972; Dicus, 1973; Palmer, 1974; Budny, 1973; Budny & McDonal, 1974) has since gone into estimating the contributions of WNC to e^+e^- annihilation. On the other hand, the only high energy e^-e^- experiment is that carried out by the Princeton–Stanford group (Princeton–Stanford Collaboration, 1966) at the modest energy of $\sqrt{(s)} \approx 1100$ MeV, using the 500 MeV electron storage rings at Stanford. It is unlikely that the WNC effects will show up at such low energies in view of the expected large mass of the Z^0. However, there have been recent proposals (PEP Project, 1974) to build higher energy ($\simeq 800$ (GeV)^2) colliding (e^-e^-) beams, and it may not be out of place to calculate the expected WNC modifications of the Møller scattering at these higher energies. One such simple calculation is reported here.

In addition to purely experimental efforts as a source of renewed interest in WNC, one has also that recent theoretical gauge models of the Weinberg–Salam type (Weinberg, 1967, 1971; Salam, 1968; Lee, 1972) require such WNC. In trying to achieve the very desirable goal of unifying electromagnetic and weak interactions and overcoming inherent divergence difficulties, one was led to a new class of interaction Lagrangians, in which WNC are manifestly present. Thus finding such WNC can lend great support to such gauge models, although it is also clear that the existence of WNC does not necessarily imply the correctness of any gauge theory. The best marriage at present between gauge theories and WNC theories is to use the form of WNC Lagrangians suggested by gauge theories for the purpose of estimating the expected magnitude of the WNC contributions to various processes. In this respect we shall use the Weinberg model as the prototype Lagrangian.

2. *Neutral Current Correction to Møller Scattering*

We now consider the Møller process $e^-e^- \to e^-e^-$ in the lowest order in which the following Feynman graphs (Fig. 1) contribute. We assume an interaction Lagrangian of the form

$$L_I = e_0 \bar{e}\gamma_\mu e A_\mu + f\bar{e}\gamma_\mu(g_V + g_A\gamma_5)eZ_\mu^0 \tag{2.1}$$

where g_V and g_A measure the relative proportions of vector and axial vector weak neutral currents; f is related to the universal Fermi coupling constant:

$$G\,(\simeq 10^{-5}\,M_N^{-2}) \qquad \text{by} \qquad f^2 = \sqrt{(2)}GM_Z^2$$

where M_Z is the mass of the intermediate vector boson and M_N the nucleon mass.

The kinematics of the reaction is as shown in Fig. 2, in which the incident electron defines the positive z axis. This electron is scattered through angle θ. All our computations are done in the centre of mass system. The two initial electrons are assumed unpolarised although, as in the case of colliding e^+e^- beams, a high degree of transverse polarisation may be achieved for the colliding e^-e^- beams, due also to synchrotron radiation (Baier, 1971). The spins of the two final electrons are summed over to obtain an unpolarised Møller cross-section. Later, we project out the spin of one of them in order to study the helicity problem. The calculations are done in the relativistic limit in which $E \gg m$, the electron mass. We then have

$$\begin{aligned}
p_{1\mu} &= (E, 0, 0, E)\\
p_{2\mu} &= (E, 0, 0, -E)\\
p'_{1\mu} &= (E, E\sin\theta\cos\phi, E\sin\theta\sin\phi, E\cos\theta)\\
p'_{2\mu} &= (E, -E\sin\theta\cos\phi, -E\sin\theta\sin\phi, -E\cos\theta)
\end{aligned} \tag{2.2}$$

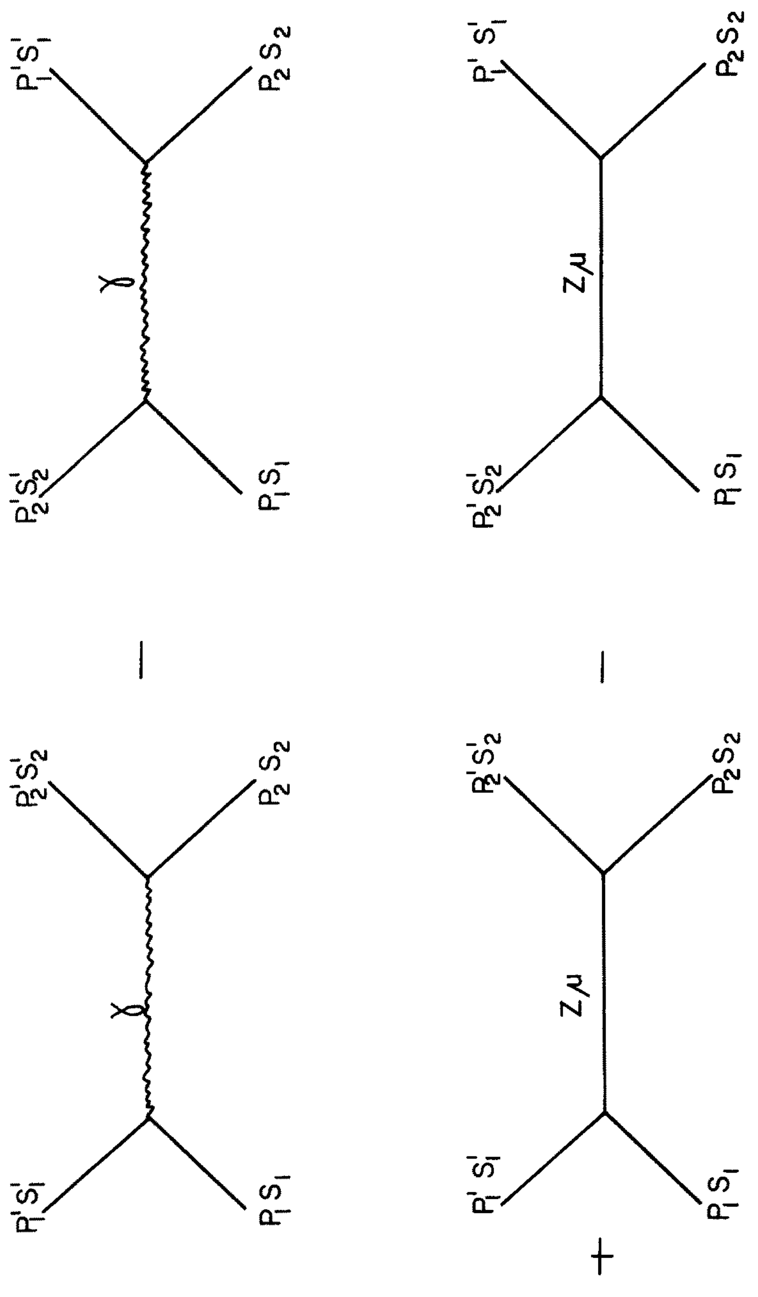

Figure 1—Lowest order Feynman graphs for Møller scattering modified by WNC.

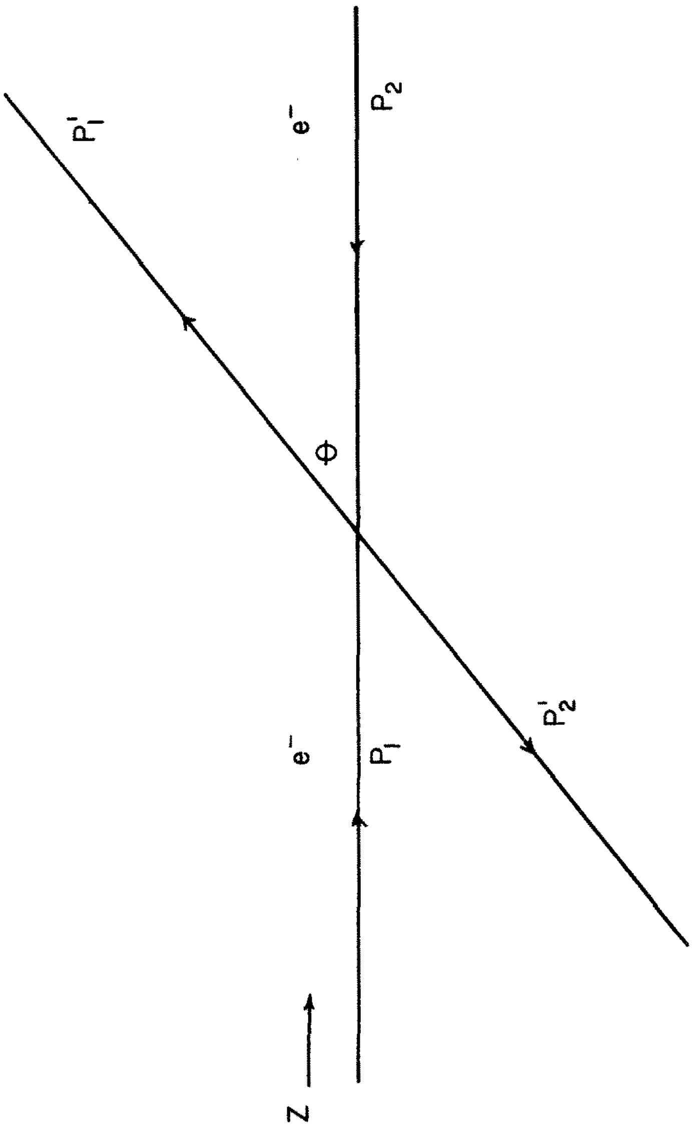

Figure 2—Centre of mass kinematics for the Møller process.

The amplitude becomes

$$M = e_0^2\bar{u}(p_1's_1')\gamma_\mu u(p_1s_1)\frac{1}{q^2}\bar{u}(p_2's_2')\gamma_\mu u(p_2s_2)$$

$$- e_0^2\bar{u}(p_2's_2')\gamma_\mu u(p_1s_1)\frac{1}{q'^2}\bar{u}(p_1's_1')\gamma_\mu u(p_2s_2)$$

$$+ f^2\bar{u}(p_1's_1')\gamma_\mu(g_V + g_A\gamma_5)u(p_1s_1)\frac{1}{q^2 - M_Z^2}\bar{u}(p_2's_2')\gamma_\mu(g_V + g_A\gamma_5)u(p_2s_2)$$

$$- f^2\bar{u}(p_2's_2')\gamma_\mu(g_V + g_A\gamma_5)u(p_1s_1)\frac{1}{q'^2 - M_Z^2}\bar{u}(p_1's_1')\gamma_\mu(g_V + g_A\gamma_5)u(p_2s_2). \tag{2.3}$$

where $q = (p_1 - p_1')$; $q' = (p_1 - p_2')$.

With the Bjorken–Drell (1964) conventions, this leads to the following expression for the amplitude squared, the two final spins being summed over

$$M^2 = X[1 + \delta_{\gamma Z} + \delta_Z] \tag{2.4}$$

where X is the high energy limit ($E^2 \gg m^2$) of the Møller scattering. We have

$$X = \frac{e_0^4}{4m^4}\left[\frac{(3 + z^2)^2}{(1 - z)^2(1 + z)^2}\right] \tag{2.5}$$

with $z = \cos\theta$. The correction terms $\delta_{\gamma Z}$ and δ_Z are the interference term and the pure weak term respectively. We have

$$\delta_{\gamma Z} = \frac{f^2E^2}{e_0^2}(1 - z^2)\frac{[8k_1 + (1 + z)^3k_2]}{\{2E^2(1 - z) + M_Z^2\}(3 + z^2)^2} + (z \to -z) \tag{2.6}$$

where $k_1 = (g_V^2 + g_A^2 + g_V^{*2} + g_A^{*2})$ and $k_2 = (g_V^2 - g_A^2 + g_V^{*2} - g_A^{*2})$.

We shall assume time reversal invariance and put $g_V^* = g_V$; $g_A^* = g_A$. The pure weak interaction term δ_Z is usually very small and can be neglected.

The spin averaged differential cross-section is given by

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{1}{4\pi^2}\frac{m^4}{4E^2}|M|^2$$
$$= \left(\frac{d\sigma}{d\Omega}\right)_M [1 + \delta_{\gamma Z} + \delta_z] \tag{2.7}$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{\alpha^2}{4E^2}\left[\frac{(3 + z^2)^2}{(1 - z)^2(1 + z)^2}\right] \tag{2.8}$$

which is the high energy limit of Møller cross-section.

For the case where we are interested in the helicity of one of the final electrons, we modify equation (2.3) by inserting the spin projection operators (Bjorken & Drell, 1964).

Taking traces as before we obtain that

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{4E^2}\left[\frac{(z^2+3)^2}{(1-z)^2(1+z)^2}\right]+\frac{\alpha f^2}{32\pi}\left\{\frac{(4k_1+(1+z)^2k_2+8hk_3)}{(1-z)[2E^2(1-z)+M_Z{}^2]}+\frac{4k_1+8hk_3}{(1-z)[2E^2(1+z)+M_Z{}^2]}+\frac{4k_1+8hk_3}{(1+z)[2E^2(1-z)+M_Z{}^2]}+\frac{4k_1+(1-z)^2k_2+4h(1+z)k_3}{(1+z)[2E^2(1+z)+M_Z{}^2]}\right\} \qquad (2.9)$$

where $k_3=(g_Vg_A+g_V^*g_A^*)$ and where we have taken the polarisation four vector of the scattered electron as

$$S_1'=\frac{h}{m}(E, p\sin\theta\cos\phi, p\sin\theta\sin\phi, p\cos\theta)$$

with h as its helicity.

From equations (2.6) and (2.7) we note that the forward-backward asymmetry (A) defined by

$$A=\frac{\frac{d\sigma}{d\Omega}(z)-\frac{d\sigma}{d\Omega}(-z)}{\frac{d\sigma}{d\Omega}(z)+\frac{d\sigma}{d\Omega}(-z)}$$

is zero. That is, the one-photon contribution as well as the electromagnetic-weak interference term is front-back symmetric. This is unlike the case of $e^+e^-\to\mu^+\mu^-$ where the axial vector part of the weak neutral current is able to give rise to an interference term which is front-back asymmetric. In that case, a search for WNC can be based on a measurement of asymmetric angular distribution. Such a measurement cannot be used for this lowest order Møller scattering. On the other hand the mean helicity $\langle h\rangle$ defined by

$$\langle h\rangle=\frac{\frac{d\sigma}{d\Omega}(h=+1)-\frac{d\sigma}{d\Omega}(h=-1)}{\frac{d\sigma}{d\Omega}(h=+1)+\frac{d\sigma}{d\Omega}(h=-1)}$$

becomes, from equation (2.9),

$$\langle h\rangle=(4f^2g_Vg_AE^2)\times\frac{4W+(1+z)(3-z)T}{R+f^2E^2\{8k_1(2E^2+M_Z^2)+\frac{1}{2}k_2[T(1-z)^3+W(1+z)^3]\}} \qquad (2.10)$$

where

$$R = \frac{e_0^2(z^2+3)^2}{1-z^2}\,[(M_Z^2+2E^2)^2 - 4E^2z^2]$$

$$T = 2E^2(1-z) + M_Z^2; \qquad W = 2E^2(1+z) + M_Z^2$$

If we put $r = E^2/M_Z^2$ and work in the limit $r \ll 1$, then equations (2.6) and (2.10) become

$$\delta_{\gamma Z} = \frac{f^2}{e_0^2}\,\frac{r(1-z^2)[8k_1 + (1+z)^3k_2]}{(3+z^2)^2} + (z \to -z) \tag{2.11}$$

$$\langle h \rangle = \frac{4f^2 g_V g_A (1-z^2) r[4 + (1+z)(3-z)]}{e_0^2(3+z^2)^2} \tag{2.12}$$

3. *Numerical Checks and Discussions*

We may now use the Weinberg model to estimate the magnitudes of these effects. According to the Weinberg model

$$g_V = -\tfrac{1}{2} + \frac{2\tan^2\alpha}{1+\tan^2\alpha}$$

$$g_A = -\tfrac{1}{2}$$

$$f = \frac{1+\tan^2\alpha}{2\tan\alpha}\, e_0$$

where α is the Weinberg angle estimated to be about 35°.

We notice that near the forward direction ($z \simeq 1$) $\delta_{\gamma Z}$ is very small, while away from the forward direction $\delta_{\gamma Z}$ can be large.

With $z = 0$, $E = 5$ GeV and $M_Z = 75$ GeV, $\delta_{\gamma Z} = 0{\cdot}5\%$. For $E \geqslant 15$ GeV, $\delta_{\gamma Z}$ can be over 4%, which is measurable. Similarly we find that $\langle h \rangle$ is small near the forward direction but can be appreciable off the forward direction. For $E = 5$ GeV and $M_Z = 75$ GeV, $\langle h \rangle \simeq -0{\cdot}1\%$. For $E \geqslant 30$ GeV, $\langle h \rangle$ is over 4%, and can be detected.

4. *Conclusion*

We conclude generally that at the proposed higher e^-e^- colliding beam energies, the Møller scattering process can be a fruitful ground for searching for weak neutral currents.

Acknowledgment

In the course of concluding this work we received a preprint (Gastmans & Van Ham, 1974) in which calculations similar to ours have been done. We are grateful to the authors for sending this preprint to us prior to publication.

References

Baier, V. N. (1971). *Uspekhi fizicheskikh nauk,* **105**, 441.

Bjorken, J. D. and Drell, S. D. (1964). *Relativistic Quantum Mechanics.* McGraw-Hill, New York.

Brown, R. W., Cung, V. K., Mikaelign, K. O. and Paschos, E. A. (1972). *Electromagnetic Background in the Search for Neutral Currents via* $e^+e^- \to \mu^+\mu^-$. Batavia preprint NAL-THY-97.

Budny, R. (1973). *Physics Letters,* **45B**, 340.

Budny, R. and McDonal, A. (1974). *Physics Letters,* **48B**, 423.

Cung, V. K., Mann, A. K. and Paschos, E. A. (1972). *Physics Letters,* **41B**, 355.

Dicus, D. A. (1973). *Physical Reviews,* **D8**, 890.

Gastmans, R. and Van Ham, Y. (1974). *Møller Scattering and Weak Neutral Currents.* University of Leuven preprint.

Godine, J. and Hankey, A. (1972). *Physical Review,* **D6**, 3301.

Hasert, F. J. *et al.* (1973). *Physics Letters,* **46B**, 138; (1973). 281; Benvenuti, A. *et al.* (1974). *Physical Review Letters,* **32**, 800.

Lee, B. W. (1972). In *Proceedings of the 16th International Conference on High Energy Physics, Batavia* (Ed. J. D. Jackson and A. Roberts).

Love, A. (1972). *Nuovo Cimento Letters,* **5**, 133.

Marshak, R. E., Raizuddin and Ryan, C. P. (1969). *Theory of Weak Interactions in Particle Physics.* Wiley–Interscience, New York.

Palmer, D. R. (1974). *Weak Neutral Currents in Polarized-Electron-Positron Storage Ring.* Toronto University preprint.

PEP Project (April 1974). SLAC Report No. 171. See also EPIC Projects in Proceedings of an Informal Meeting on Links between Weak and Electromagnetic Interactions. Rutherford Laboratory Report No. RL-73-018, 1973.

Princeton–Stanford Collaboration (1966). *Physical Review,* **16**, 1127 (see also S. J. Brodsky in Proceedings of the 4th International Symposium on Electron and Photon Interactions at High Energies, Daresbury Laboratory, 1969).

Salam, A. (1968). *Proceedings of the 8th Nobel Symposium, Stockholm* (Ed. N. Svartholm

Weinberg, S. (1967). *Physical Review Letters,* **19**, 1264; (1971). **27**, 1688.